Medium Effects on Heavy-Atom Kinetic Isotope Effects. II. Cell Model with External–External and Certain External–Internal Coordinate Interactions

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Abstract: The method of Stern, Van Hook, and Wolfsberg for the calculation of vapor pressure isotope effects was applied to model calculations of medium effects on ^{13}C kinetic isotope fractionation in the decompositions (*via* several reaction coordinates) of a hypothetical nonlinear triatomic; molecular parameters were given values characteristic of organic compounds, while the necessary external force constants were adjusted to yield associated frequencies typical of moderate H bonding (30–90 cm^{-1}). Influences of interactions between external coordinates were studied, as were those between external coordinates and an internal coordinate which was not a component of the reaction coordinate. TAM was selected as the model because of its small size, all such interaction effects decreasing rapidly with increasing molecular mass and complexity. In no case studied would the medium effects calculated be measurable given present techniques; indeed, for molecules of ordinary mass and complexity, such effects would not even be a detectable increment or decrement to isotope rate effects determined with reasonable precision. Because the effects are additive, subject to certain restrictions, it is possible that medium effects of experimental significance could be produced *if:* the isotopic molecules were relatively light; reaction temperatures were low (say, below 25°); solvent-solute interactions were drastically different in magnitude in reaction and transition states; one or more of the component interactions includes coupling to an internal coordinate (preferably isotopic) which is part of the reaction coordinate; and the force constants associated with such interactions are substantially larger than those employed here. If the last two conditions are met, the assumptions of the cell model will have been strained and some more natural technique for handling such strong interactions should be attempted.

The first report in this series[1] dealt with an application, to calculation of medium effects on heavy-atom kinetic isotope effects, of the technique developed and used for study of vapor pressure isotope effects by Stern, Van Hook, and Wolfsberg.[2] Simple decomposition reactions of a hypothetical three-atom nonlinear molecule (TAM), formic acid, and oxalic acid were investigated. The external force constants required for this cell model approach were adjusted so that the additional frequencies generated lay in the 30–90-cm^{-1} range characteristic of moderate hydrogen bonding. In that study just medium–molecule interactions were investigated: no interactions were introduced among external coordinates or between external and internal coordinates, nor was any external coordinate incorporated into the reaction coordinates assumed.

The medium-related isotope fractionations, found in the previous calculations, were linearly related to the force constant shifts employed, were found to be additive and to vary inversely with the $^{2}/_{3}$ power of molecular mass, but were uniformly so small as to be negligible in comparison with ordinary imprecision in isotope rate effect measurements.

This paper reports the next step in the investigation of medium effects using the cell model. Calculations were carried out only on the TAM model, described in Table I. Check computations showed the $M^{-2/3}$ dependence to obtain, so effects on TAM were expected to be the largest among the three test molecules. We studied the results of interactions between different pairs of external coordinates and between one internal coordinate and a number of different individual external coordinates on ^{13}C kinetic isotope effects.

Table I. Values of Input Parameters, TAM[a–c]

Internal coordinate	No. i	$F_{i,i}$[e]
B–C stretch[d]	1	4.4
A–B stretch	2	4.4
A–B–C angle bend	3	1.63
External force constants[e,f]	φ_3	
F_{T_x}	0.16	
F_{T_y}	0.16	
F_{T_z}	0.16	
F_{R_x}	0.0196	
F_{R_y}	0.0188	
F_{R_z}	0.02	

[a] Masses are in atomic mass units. [b] Atomic masses: A, 12.0; B, 12.0; C, 12.0/13.0. [c] Bond distances (ångstroms): A–B, 1.54; B–C, 1.54; the bond angle is tetrahedral. [d] C is ^{13}C in the heavy species. [e] Stretching force constants are in mdyn/Å; bend force constant in mdyn Å. Translational force constants are in mdyn/Å; rotational force constants are in mdyn Å. [f] These external force constants comprise force field φ_3 of ref 1. The molecule was defined in the x,y plane.

Computation Methods

All calculations were carried out with the framework of transition state theory using a local modification of Schachtschneider's programs[3] for solution of the Wilson **GF** matrix problem.[4]

The method of Stern, Van Hook, and Wolfsberg requires that six external coordinates be defined (three translations and three rotations), and they are not mass

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(2) M. J. Stern, W. A. Van Hook, and M. Wolfsberg, *J. Chem. Phys.*, 3179 (1963).

(3) J. H. Schachtschneider and R. G. Snyder, *Spectrochim. Acta*, **19**, 177 (1965).

(4) E. B. Wilson, Jr., J. C. Decius, and P. C. Cross, "Molecular Vibrations," McGraw-Hill, New York, N. Y., 1955.

Table II. Elements $G_{i,j}$ of $\mathbf{G}_\alpha$ (Upper Triangle) and $\mathbf{G}_\beta$ (Lower Triangle); Nonlinear TAM

	i									
j	1	2	3	T_x	T_y	T_z	R_x	R_y	R_z	
	0.166667	−0.027794	−0.051015	0.0	0.0	0.0	0.0	0.0	0.0	1
		0.166667	−0.051015	0.0	0.0	0.0	0.0	0.0	0.0	2
1	0.160256		0.163995	0.0	0.0	0.0	0.0	0.0	0.0	3
2	−0.027794	0.166667		0.027778	0.0	0.0	0.0	0.0	0.0	T_x
3	−0.051015	−0.051015	0.161292		0.027778	0.0	0.0	0.0	0.0	T_y
T_x	−0.001745	0.0	−0.000801	0.027066		0.027778	0.0	0.0	0.0	T_z
T_y	−0.001233	0.0	0.001133	0.0	0.027066		0.158172	0.0	0.0	R_x
T_z	0.0	0.0	0.0	0.0	0.0	0.027066		0.026350	0.0	R_y
R_x	0.0	0.0	0.0	0.0	0.0	−0.001200	0.156143		0.022587	R_z
R_y	0.0	0.0	0.0	0.0	0.0	0.000850	0.001434	0.025337		
R_z	−0.000841	0.0	0.001352	0.000171	−0.000728	0.0	0.0	0.0	0.021801	

independent as are all the internal coordinates.[2,5,6] As is customary, we selected the light isotopic species (α) as the basis in forming the external coordinates of the heavy species (β). Examination of Table II shows $\mathbf{G}_\alpha$ to have only diagonal external elements and no external–internal cross terms such as appear in $\mathbf{G}_\beta$.

As in the earlier work, ground and transition state geometries were identical. Transition state force fields, $\mathbf{F}^{\ddagger}$, differed from that of the ground state, $\mathbf{F}^0$, only as required by the technique for generating the preselected reaction coordinate. In one approach (type I calculations), $F_{1,1}^{\ddagger}$ or $F_{2,2}^{\ddagger}$ was set to zero, resulting in reaction coordinate eigenvectors, $\mathbf{L}$, consisting of one nonzero element, L_1 or L_2, respectively; in the other (type II calculations), $F_{1,2}^{\ddagger} = F_{2,1}^{\ddagger}$ was set to the − or + value of the geometric mean of $F_{1,1}^{\ddagger} \equiv F_{1,1}^0$ and $F_{2,2}^{\ddagger} \equiv F_{2,2}^0$, resulting in reaction coordinate eigenvectors consisting of two nonzero elements and corresponding to symmetric ($L_1 = \pm$, $L_2 = \pm$) or asymmetric ($L_1 = \pm$, $L_2 = \mp$) relative motion in internal coordinates 1 and 2. (For brevity, these four reaction coordinates will be designated 1, 2, 12+, and 12−.) Both of these approaches yield a reaction coordinate eigenvalue, λ_1, of zero; that is, $\lambda_1 = 4\pi^2(\nu_1^{\ddagger})^2 = 0$, and the assumed potential barrier is flat.

These simple techniques for producing a transition state place restrictions on the construction of $\mathbf{F}^{\ddagger}$ which limit the kinds of tests one can make of medium effects, especially on a model as simple as TAM. In a type I calculation, where $\mathbf{L} = L_i$ and $\nu_1^{\ddagger} = 0$, there can be no nonzero elements in the ith row or column of $\mathbf{F}^{\ddagger}$. In a type II calculation, where $\mathbf{L} = (L_i,L_j)$ and $\nu_1^{\ddagger} = 0$, the only nonzero elements in rows (or columns) i and j of $\mathbf{F}^{\ddagger}$ are $F_{i,i}^{\ddagger}$, $F_{j,j}^{\ddagger}$, and $F_{i,j}^{\ddagger}$. Because of these restrictions, one cannot include in either technique employed here and in the earlier study[1] a nonzero interaction force constant between any pair of coordinates, one of which is an element of the reaction coordinate, and retain comparability of different sets of results. Of course, such cross terms can actually be included in $\mathbf{F}^{\ddagger}$ (or even $\mathbf{F}^0$), but the simplicity of the reaction coordinates is lost.

The work of Ishida and Bigeleisen[7] on the vapor pressure isotope effect, and consideration of symmetry relationships among the internal and external coordinates involved in the construction of the matrices $\mathbf{G}_\alpha$ and $\mathbf{G}_\beta$, leads one to expect that interaction force constants coupling an internal and an external coordinate of the same symmetry will generate effects much larger than those which couple coordinates of different symmetry. $\mathbf{G}_\alpha$ in the cell model has no internal–external elements, so one must look to $\mathbf{G}_\beta$ for such symmetry relationships. The matrix $\mathbf{G}_\beta$ for TAM describes a species belonging to point group C_s. Reference to the character table[8] reveals that the translations T_x and T_y, and the rotation R_z, are of the same symmetry as the internal coordinates, while T_z, R_x, and R_y are not. The only internal coordinate which can be involved in internal–external coordinate interactions (for $\nu_1^{\ddagger} = 0$ and within the restrictions of the type I and type II calculation methods) is 3, the angle bend. Hence the largest relative medium effects are expected to be those involving nonzero values of the interaction force constants F_{3,T_x}, F_{3,T_y}, and F_{3,R_z}.

Results and Discussion

Interactions among External Coordinates. External–external interaction force constants are limited in magnitude to values less than the geometric mean of the related diagonal force constants. That is

$$|F_{x,y}| < |(F_{x,x}F_{y,y})^{1/2}| \equiv |(F_{x,y})_0| \qquad (1)$$

If $|F_{x,y}| \geq |(F_{x,y})_0|$, a nongenuine vibration will result; in the transition state this nongenuine mode is in addition to the reaction coordinate.

Table III summarizes the values of $L(k/k') = L\cdot(k_{^{12}\mathrm{C}}/k_{^{13}\mathrm{C}})$, $L(x) = 100 \ln x$, for TAM reacting *via* the four reaction coordinates described above. $L(k/k')$ is the sum of two terms: $L(\mathrm{TIF}) = L(\nu_1^{\ddagger}/\nu_1'^{\ddagger})$, which arises in the motion in the reaction coordinate and is temperature independent; and a temperature dependent factor $L(\mathrm{TDF})$ which arises in isotopic shifts of genuine vibrational frequencies between reactant and transition states. Data are shown for near-limit values of the three interaction force constants which couple pairs of external coordinates belonging to the same symmetry class as the internal coordinates. As is expected from the discussion above, near-limit values of force constants such as F_{T_z,T_z}, F_{R_y,R_y}, etc., which couple coordinates either of which has symmetry different from that of the internal coordinates, were without effect on $L(k/k')$ within round-off error of our double precision computations.

The results of Table III were obtained by employing the same medium interaction with the reactant and transition states; neither the diagonal external nor the

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(6) M. Wolfsberg, *J. Chim. Phys.* **60**, 15 (1963).
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(8) D. S. Schonland, "Molecular Symmetry," Van Nostrand, London, 1965.

Table III. Calculated Values of $L(k/k')$[a,b]

T, deg C	Reaction coordinate 1	2	12+	12−
A. No External–External Interaction				
12.55	6.18758	0.39747	2.70462	1.35733
60.17	5.42933	0.38906	2.44942	1.21437
90.20	4.97053	0.38545	2.30529	1.13362
126.84	4.70842	0.38386	2.22761	1.09022
L(TIF)	2.61427	0.37980	1.85033	0.88490
B. $F_{T_x,T_y} = 0.159000$; $(F_{T_x,T_y})_0 = 0.160000$				
12.55	6.18780	0.39748	2.70470	1.35738
60.17	5.42946	0.38906	2.44946	1.21440
99.20	4.97062	0.38545	2.30532	1.13364
126.84	4.70850	0.38386	2.22763	1.09023
C. $F_{T_x,R_z} = 0.055000$; $(F_{T_x,R_z})_0 = 0.056569$				
12.55	6.18763	0.39747	2.70464	1.35734
60.17	5.42936	0.38906	2.44943	1.21438
99.20	4.97055	0.38545	2.30530	1.13363
126.84	4.70844	0.38386	2.22761	1.09023
D. $F_{T_y,R_z} = 0.055000$; $(F_{T_y,R_z})_0 = 0.056569$				
12.55	6.18761	0.39761	2.70463	1.35734
60.17	5.42935	0.38906	2.44942	1.21438
99.20	4.97054	0.38545	2.30529	1.13363
126.84	4.70843	0.38386	2.22761	1.09022

[a] $L(x) = 100 \ln x$. [b] Same external–external interaction force constant for the reactant and transition state.

off-diagonal external–external force constants differed between these states.

Values of L(TIF) shown in Table IIIA are slightly larger than the corresponding values for reactions in the gas phase. Since all results to be discussed here are for reactions in a condensed phase, and since L(TIF) is unaffected by any of the $\mathbf{F}^0$ or $\mathbf{F}^{\ddagger}$ manipulations employed, these data are not repeated in later tabulations.

Comparison of $L(k/k')$ values in part A of Table III with corresponding data in parts B, C, and D shows the effects of different single external–external interactions on L(TDF); though all are very small, the effects are similar in magnitude to the gross medium effects (produced by changes in diagonal external force constants only) described in the previous publication.[1] These effects differ among the several reaction coordinates, and that pattern too is similar to that of the "no coordinate interactions" results reported earlier. In every case, however, these medium effects are completely negligible in comparison to ordinary experimental error.

It might be argued that larger and perhaps significant external medium effects could be produced by assuming that both components (diagonal and interaction) were drastically different in the media surrounding the reactant and transition states. The values of the external–external interaction elements of **F** are limited by those of the corresponding diagonal elements, eq 1. The related isotopic shifts in **G**, Table II, are too small to yield significant medium effects at the level of the external elements in $\mathbf{F}^0$ and $\mathbf{F}^{\ddagger}$ employed here. We show later that these single self-interactions of the medium yield effects much smaller than other likely interactions. Accordingly multiple self-interaction effects are also likely to be very small.

It is worth noting that small but nonzero medium effects are generated by interaction force constants for which the related $G_\alpha - G_\beta$ are zero. This observation indicates that even these very small fractionation shifts involve higher than first-order terms in the high-temperature approximation.[9,10]

Table IV. Calculated Values of $L(k/k')$[a]

T, deg C[b]	Reaction coordinate 1	2	12+	12−
A. No External–Internal Interaction (see Table IIIA)				
B. $F_{3,T_x} = 0.510000$; $(F_{3,T_x})_0 = 0.510686$				
12.55	6.21461	0.40038	2.73244	1.34404
60.17	5.44617	0.39054	2.46665	1.20496
99.20	4.98231	0.38633	2.31731	1.12647
126.84	4.71771	0.38448	2.23706	1.08430
C. $F_{3,T_x} = 0.255000$				
12.55	6.20109	0.39891	2.71851	1.35069
60.17	5.43775	0.38979	2.45802	1.20967
99.20	4.97641	0.38589	2.31129	1.13005
126.84	4.71306	0.38416	2.23233	1.08726
D. $F_{3,T_y} = 0.510000$; $(F_{3,T_y})_0 = 0.510686$				
12.55	6.20493	0.39782	2.72107	1.34942
60.17	5.44038	0.38927	2.46002	1.20857
99.20	4.97836	0.38559	2.31286	1.12911
126.84	4.71464	0.38396	2.23364	1.08644
E. $F_{3,T_y} = 0.255000$				
12.55	6.19625	0.39763	2.71282	1.35338
60.17	5.43485	0.38916	2.45470	1.21147
99.20	4.86623	0.38551	2.30907	1.13137
126.84	4.71153	0.38390	2.23062	1.08833
F. $F_{3,R_z} = 0.175000$; $(F_{3,Z_z})_0 = 0.180555$				
12.55	6.19145	0.39734	2.70810	1.35564
60.17	5.43182	0.38901	2.45171	1.21311
99.20	4.97230	0.38542	2.30695	1.32163
126.84	4.70984	0.38384	2.22289	1.08938
G. $F_{3,R_z} = 0.087500$				
12.55	6.18951	0.39741	2.70636	1.35649
60.17	5.43057	0.38903	2.45056	1.21374
99.20	4.97141	0.38544	2.30161	1.13313
126.84	4.70913	0.38385	2.22828	1.08980

[a] Same external–internal interaction force constant for the reactant and transition state. [b] These temperatures, which are arbitrary, lie in the range $\theta = 10001T$°K $= 3.50$–2.50, convenient for experimentation with many common solvents.

Interaction of Internal Coordinate 3 and External Coordinates. Table IV compiles medium-related changes in $L(k/k')$ for different levels of each of the three internal–external coordinate interactions for which nonzero effects were expected and observed. The same interaction was employed with reactant and transition states. These single interaction effects are linearly related to the force constant value, and their substantial size in comparison with external–external interaction effects is ascribable to a larger force constant; in absolute magnitude they are, of course, also so small as to be negligible.

Combination of several of these single interaction effects might be thought to yield a sum of greater significance. Where there is, as 3 here, a common coordinate in all the external–internal interaction force constants, their values are subject to the restriction[11]

$$(\alpha_{3,i})^2 + (\alpha_{3,j})^2 + \ldots < 1 \tag{2}$$

where

$$\alpha_{3,i} = F_{3,i}/|(F_{3,3}F_{i,i})^{1/2}| \tag{3}$$

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[9] J. Bigeleisen, *Proc. Int. Symp. Isotope Separ. 1st., 1957* (1958).

[10] M. Wolfsberg and M. J. Stern, *Pure Appl. Chem.*, **8**, 225 (1964).

[11] T. T.-S. Huang, W. J. Kass, W. E. Buddenbaum, and P. E. Yankwich, *J. Phys. Chem.*, **72**, 4431 (1968).

That is, the effect on $L(k/k')$ of several $F_{n,i}$, coupling in reaction or transition states or both a common (here internal) coordinate n with several other coordinates i, j, etc., cannot exceed the largest effect of an individual $F_{n,i}$. If the condition of eq 2 is violated, a nongenuine vibration appears: a second in the transition state, a first in the reactant.

Comparisons of the data in Tables III and IV and certain findings in the preceding study in this series[1] suggest that substantial medium effects can be produced if different internal–external interaction manifolds apply to the reactant and transition states, that is, if the striction between medium and molecule differs between these two states. Within computer round-off error, all interaction effects recorded previously are linear and, except for the restricted types of combination noted above, are additive.

Let $\varphi_A{}^0$ represent the A level of external–external and/or external–internal coordinate interactions in F^0; similarly, the B level of such force constants in $F^{\ddagger}$ would be $\varphi_B{}^{\ddagger}$. If $(M)^0$ and $(M)^{\ddagger}$ represent the reactant and transition states of TAM, then the data of Tables III and IV are for processes and strictions which may be written

$$\begin{aligned} (M)^0\varphi_A{}^0 &\rightleftharpoons (M)^{\ddagger}\varphi_A{}^{\ddagger} \xrightarrow{k} \text{products} \\ (M')^0\varphi_A{}^0 &\rightleftharpoons (M')^{\ddagger}\varphi_A{}^{\ddagger} \xrightarrow{k'} \text{products} \end{aligned} \qquad (4)$$

and for which the kinetic isotope effect may be written $L(k/k')_{A^0A^{\ddagger}}$. Let $(K_{eq})_{A^0B^0}$ be the equilibrium constant for the hypothetical isotope exchange reaction

$$(M)^0\varphi_B{}^0 + (M')^0\varphi_A{}^0 \rightleftharpoons (M)^0\varphi_A{}^0 + (M')^0\varphi_B{}^0 \qquad (5)$$

Then, the effects of differences in reactant–transition state striction manifolds may be written

$$L(k/k')_{B^0A^{\ddagger}} = L(k/k')_{A^0A^{\ddagger}} + L(K_{eq})_{A^0B^0} \qquad (6)$$

Table V is a compilation of equilibrium constant values for interaction of internal coordinate 3 with each of the external coordinates, m. Uniformly, $(K_{eq})_{A^0B^0}$ is calculated for an *increase* in reactant striction in comparison to that effective in the transition state. Where $G_\alpha - G_\beta \neq 0$, the entries in Tables I and II account well for both the signs and magnitudes of the tabulated K_{eq}; the examples are F_{3,T_x}, F_{3,T_y}, and F_{3,R_z}. The nonzero and nonlinear effects when $G_\alpha - G_\beta = 0$, *i.e.*, for F_{3,T_z}, F_{3,R_x}, and F_{3,R_y}, arise in terms higher than first order of the high-temperature approximation.

One can imagine transition states in which, say, polarity differences from the reactant state are so large as to affect solvent–solute interactions. The data of Table V show that where these differences are so great that something like moderate internal–external interactive hydrogen bonding in one state almost disappears in the other a medium isotope effect of possibly *detectable* magnitude can result, especially at low temperatures. For example, the sum over both columns of all the effects at 12.55° (*viz.*, a sum of the effects, eq 4, such as $[A^0 \rightarrow B^0] + [B^0 \rightarrow C^0]$) is equivalent to a shift in value of $L(K_{eq})_{A^0B^0}$ of about 0.09 (or a tenth of a per cent). It seems doubtful that one could *measure* an effect like this given present techniques.

Conclusions

All of the interactions studied here and in the previous investigation are weak; force constants were selected so that the vibrational frequencies associated with medium–molecule interactions in the cell model would be at the level characteristic of moderate hydrogen bonding. At such $F_{i,j}$ levels, all medium–medium and medium–molecule interactions which involve coupling of coordinates *not elements of the reaction coordinate* have negligible effects on heavy-atom kinetic isotope effects except at very low temperatures when the reaction is of a very light (and simple) molecule. External–internal coordinate interactions yield effects much larger than those among external coordinates.

Attempts to explain experimental results by invoking appreciable medium–molecule interactions arising (in the cell model) in external–internal interactions substantially larger than those employed here could be successful were the magnitude of $L(k/k')$ of principal concern but not its temperature dependence; the contributions of medium effects to $L(k/k')$ have very simple temperature dependence (Stern's class A).[12,13] In such attempts, one would have to seek independent chemical or physical justification for the large values of the force constants employed, and one would also have to confront the very real problem of interactions between external coordinates and one or more internal

Table V. Calculated Values of $L(K_{eq})_{A^0B^0}$ for Different Levels (A^0 and B^0) of Single External–Internal Coordinate Interactions[a]

	A. Translations	
T, deg C	$\varphi_A{}^0$: $F_{3,m} = 0.0$ $\varphi_B{}^0$: $F_{3,m} = 0.255$	$\varphi_A{}^0$: $F_{3,m} = 0.255$ $\varphi_B{}^0$: $F_{3,m} = 0.510$
	$m = T_x$	
12.55	−0.07495	−0.07275
60.17	−0.05545	−0.05417
99.20	−0.04450	−0.04363
126.84	−0.03856	−0.03789
	$m = T_y$	
12.55	0.08094	0.08326
60.17	0.06259	0.06393
99.20	0.05177	0.05266
126.84	0.04569	0.04637
	$m = T_z$	
12.55	0.00113	0.00338
60.17	0.00066	0.00197
99.20	0.00044	0.00132
126.84	0.00034	0.00102
	B. Rotations	
T, deg C	$\varphi_A{}^0$: $F_{3,m} = 0.0$ $\varphi_B{}^0$: $F_{3,m} = 0.0875$	$\varphi_A{}^0$: $F_{3,m} = 0.0875$ $\varphi_B{}^0$: $F_{3,m} = 0.1750$
	$m = R_x$	
12.55	0.00052	0.00156
60.17	0.00030	0.00091
99.20	0.00020	0.00061
126.84	0.00016	0.00047
	$m = R_y$	
12.55	0.00017	0.00050
60.17	0.00010	0.00029
99.20	0.00006	0.00019
126.84	0.00005	0.00015
	$m = R_z$	
12.55	0.03451	0.03478
60.17	0.02650	0.02665
99.20	0.02181	0.02191
126.84	0.01919	0.01927

[a] The diagonal internal and external force constants are unchanged between the states A^0 and B^0.

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(12) M. J. Stern, W. Spindel, and E. U. Monse, *J. Chem. Phys.*, **48**, 2908 (1968).

(13) P. C. Vogel and M. J. Stern, *J. Chem. Phys.*, **54**, 779 (1971).

coordinates which were elements of the reaction coordinate.

Two recent surveys[14,15] of the influences of internal–internal coordinate coupling on calculated kinetic isotope effects suggest that inclusion of an element of the reaction coordinate (especially a bond stretch) in such a couple could lead to effects substantially larger than those of bend–external coordinate interaction described here. First, the larger size of diagonal stretching force constants would increase the magnitude of the coupling force constant; second, modification of the restrictions on type I and type II calculations, to retain the preselected zero eigenvalue but permit additional, though minor, elements in the reaction coordinate eigenvector, would result in shifts in $F_{i,i}^{\ddagger}$ or $F_{i,j}^{\ddagger}$, which would affect $L(k/k')$ through terms $G_\alpha - G_\beta$ which are intrinsically large. (With or without these embellishments, anharmonicity corrections to $L(k/k')$ may be neglected.[16])

There are other attractive possibilities for the description and calculation of such strong medium interactions. One of these, the structured medium model, is under active investigation in our laboratory.

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(16) Corrections for anharmonicity properly made[17] have resolved discrepancies between theory and experiment in a number of H/D isotopic exchange equilibria;[18] the same principles would be expected to apply here (see eq 6 and Table V). Such corrections decrease rapidly with increasing principal moments of inertia and should be entirely negligible for ^{13}C isotope effects which have a dynamic as well as an equilibrium component (as here). For, as Van Hook has said,[19] "In the case of isotope rate effects, the inclusion of anharmonic correction terms seems a little pointless for the usual calculation where the parameters defining the transition state are essentially guessed." In the context of this report the isotope rate effect is primary and may be large, the medium effects are secondary and very small, and anharmonicity corrections to either may safely be neglected.

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Cis Double Bonds in Liquid Crystalline Bilayers[1]

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Abstract: The structure of lipid bilayers composed of cis unsaturated fatty acids is investigated by means of spin labels. It is found that the hydrocarbon chains assume a bent configuration and impose a certain degree of order even on the central part of the bilayer. Furthermore the spin labels indicate a rapid anisotropic motion of the bent lipid molecules with a correlation time of approximately 3×10^{-10} sec, the axis of motional averaging being perpendicular to the bilayer normal in the range of 30 to 80°. Below 30° the axis of motional averaging changes and the hydrocarbon chains are tilted with respect to the bilayer normal. From the flexibility gradient of the hydrocarbon chains a parameter characteristic of the interaction energy can be extracted. In bilayers with cis double bonds the molar interaction energy is found to be 300 cal per CH_2 unit smaller as compared to bilayers with saturated chains. Spin label data and X-ray diffraction measurements reveal a remarkable temperature adaptability of the unsaturated lipid bilayers. The linear expansion coefficient amounts to only -0.5×10^{-3} °K^{-1}.

Lyotropic liquid crystals are suitable model systems for investigating structural properties of lipid membranes. Spin label studies have provided information about the structure of the various lyotropic phases and about the mobility in these phases.[3–8] In earlier communications we reported spin label results of lyotropic mesophases containing lipid molecules with saturated hydrocarbon chains. In this work we present results pertinent to the influence of cis double bonds using multilamellar systems of potassium oleate or potassium petroselinate and water, which we investigate with spin labeled fatty acids to probe the conformation of the hydrocarbon chains. The lipid layers of these systems are thus composed of identical molecules, all having a cis double bond between carbon atoms 9 and 10 (oleic acid) or 6 and 7 (petroselinic acid) which facilitates the molecular interpretation of the experimental results. Experimentally these systems have the further advantage that they can be oriented homogeneously which simplifies the spectra and allows an accurate quantitative evaluation. Since cis unsaturated fatty acids are essential elements for the proper functioning and the structure of many biological membranes, the purpose of this investigation is to contribute to a better understanding of the role of cis double bonds in biological systems.

Materials and Methods

Liquid Crystalline Phase. The phase diagram of the binary mixture potassium oleate–water (KOW) at 20°

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